From Two Loops to a Matrix

A guided Kirchhoff and Jupyter bridge

Student number and last name: _____

Class or period: _____

Learning goals

By the end of this bridge, I can:

- define the three current directions in a two-loop circuit;
- write one KVL equation for each loop and handle the shared resistor;
- rearrange both equations into the same variable order;
- build $A\mathbf{x} = \mathbf{b}$ and solve it numerically and symbolically;
- verify the result with KCL, KVL, and a residual.

Why this is useful

Two loops share one component, so the current through that component depends on both loop currents. A matrix does not replace Kirchhoff's laws. It organizes the equations after the circuit reasoning is complete.

What you will make

You will submit one notebook showing this complete chain:

arrows $\longrightarrow$ two KVL equations $\longrightarrow$ standard form $\longrightarrow$ $A\mathbf{x} = \mathbf{b}$ $\longrightarrow$ two solver methods $\longrightarrow$ checks

Before you start

- Use Question 4 as the worked model, then transfer the method to Question 5.
- Keep the released Question 5 answer closed until your matrix rows read back correctly.
- Download the course notebook from the assignment page.
- Keep resistance in $\mathrm{k}\Omega$, voltage in V, and current in mA.
- This checkpoint is a computational verification activity. No physical lab is required.

Textbook support: Grob, Chapter 9, pp. 264–281. **Practice support:** Problems Manual, Chapter 9.
Engagement review: ElectroBOOM101-008: Kirchhoff Circuit Laws.

Start with current directions and the shared branch

1 Define the variables before writing equations

For the completed Question 4 circuit, use clockwise mesh currents I_1 and I_2. The two mesh currents move through the shared resistor in opposite directions. Define I_3 downward through the shared resistor.

$$I_1 - I_2 - I_3 = 0 \quad \Longrightarrow \quad I_3 = I_1 - I_2$$

This is KCL at the shared branch. It fixes the meaning of every later sign.

I_1	clockwise current assigned to Loop 1
I_2	clockwise current assigned to Loop 2
I_3	downward current through R_3, defined as $I_1 - I_2$

2 Use one sign rule from start to finish

As you walk around a loop:

Source from $-$ to $+$	voltage rise: write $+V_S$
Source from $+$ to $-$	voltage drop: write $-V_S$
Resistor in the current direction	voltage drop: write $-IR$
Resistor against the current direction	voltage rise: write $+IR$

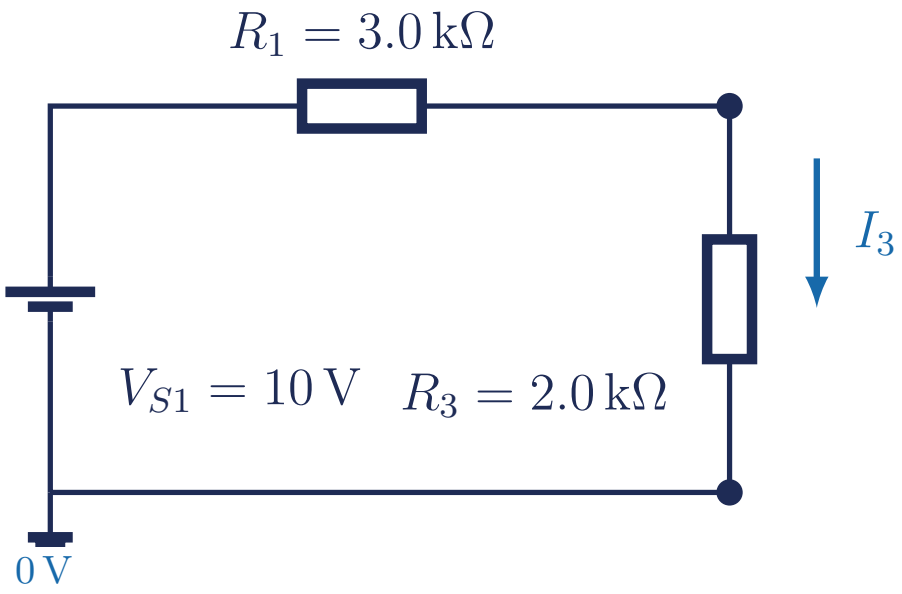
Essential skill: do not jump directly to the matrix. First write the signed KVL terms in the order encountered around each loop. Then substitute $I_3 = I_1 - I_2$.

Quick check before continuing

If the final value of I_3 is negative, what would that mean?

Loop 1: write, substitute, expand, and collect

3 Walk clockwise around Loop 1



Element	Signed KVL term	Reason
$V_{S1} = 10\,\text{V}$	$+10$	rise through the source
$R_1 = 3.0\,\text{k}\Omega$	$-3I_1$	travel with I_1
$R_3 = 2.0\,\text{k}\Omega$	$-2I_3$	travel with the defined I_3

Write the voltage changes before substituting Ohm's law:

$$+10 - V_{R1} - V_{R3} = 0$$

Replace each resistor voltage with IR:

$$V_{R1} = I_1 R_1 = 3I_1, \qquad V_{R3} = I_3 R_3 = 2I_3$$

$$+10 - 3I_1 - 2I_3 = 0$$

Substitute the shared current:

$$+10 - 3I_1 - 2(I_1 - I_2) = 0$$

Distribute -2 to both terms in the bracket:

$$+10 - 3I_1 - 2I_1 + 2I_2 = 0$$

Collect the two I_1 terms:

$$+10 - 5I_1 + 2I_2 = 0$$

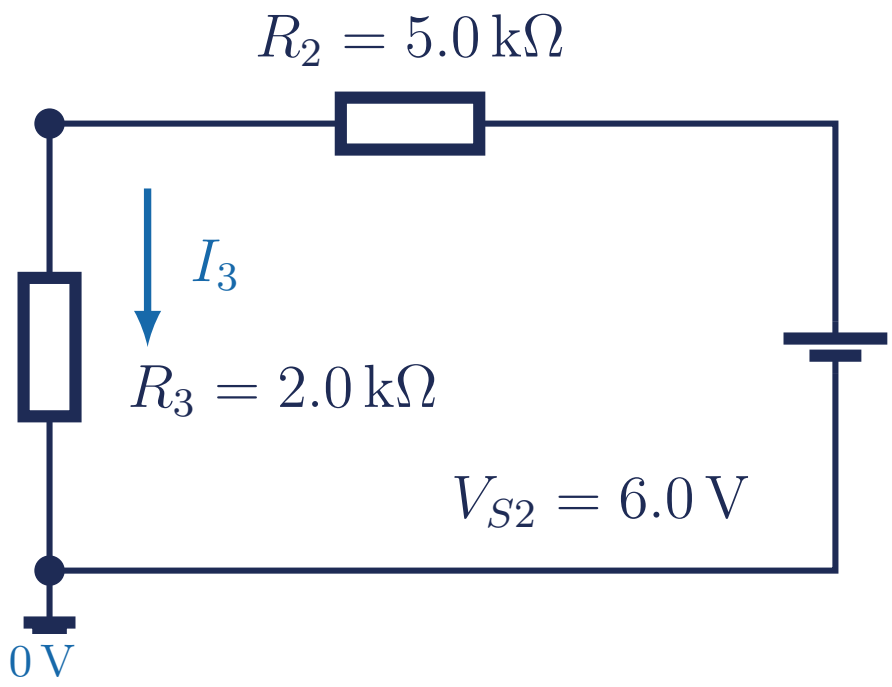
Subtract 10 from both sides, then multiply every term by -1:

$$-5I_1 + 2I_2 = -10 \quad \Longrightarrow \quad \boxed{5I_1 - 2I_2 = 10}$$

The first matrix row is read directly from the standard-form equation: coefficient of I_1, coefficient of I_2, right side. Therefore Row 1 is $[5, -2]$ and $b_1 = 10$.

Loop 2: the shared resistor changes sign

4 Walk clockwise around Loop 2



Element	Signed KVL term	Reason
$V_{S2} = 6.0\,\text{V}$	$+6$	rise through the source
$R_2 = 5.0\,\text{k}\Omega$	$-5I_2$	travel with I_2
$R_3 = 2.0\,\text{k}\Omega$	$+2I_3$	travel against the defined I_3

Write the voltage changes before substituting Ohm's law:

$$+6 - V_{R2} + V_{R3} = 0$$

Replace each resistor voltage with IR:

$$V_{R2} = I_2 R_2 = 5I_2, \qquad V_{R3} = I_3 R_3 = 2I_3$$

$$+6 - 5I_2 + 2I_3 = 0$$

Substitute the shared current:

$$+6 - 5I_2 + 2(I_1 - I_2) = 0$$

Distribute $+2$ to both terms in the bracket:

$$+6 - 5I_2 + 2I_1 - 2I_2 = 0$$

Reorder the current terms, then combine the I_2 terms:

$$+6 + 2I_1 - 5I_2 - 2I_2 = 0 \quad \Longrightarrow \quad +6 + 2I_1 - 7I_2 = 0$$

Subtract 6 from both sides:

$$2I_1 - 7I_2 = -6$$

Multiply the complete equation by -1:

$$\boxed{-2I_1 + 7I_2 = 6}$$

The second matrix row is $[-2,\ 7]$ and $b_2 = 6$. Multiplying a complete equation by -1 does not change its solution.

Put both equations into one matrix

5 Align the variable order

Use the same order, $[I_1, I_2]$, in every equation and every matrix row.

$$\begin{aligned} &\text{Equation 1:} \quad 5I_1 - 2I_2 = 10 \\ &\text{Equation 2:} \quad -2I_1 + 7I_2 = 6 \end{aligned}$$

	coefficient of I_1	coefficient of I_2	right side
Equation 1	5	-2	10
Equation 2	-2	7	6

$$\underbrace{\begin{bmatrix} 5 & -2 \\ -2 & 7 \end{bmatrix}}_{A\ [\mathrm{k\Omega}]} \underbrace{\begin{bmatrix} I_1 \\ I_2 \end{bmatrix}}_{\mathbf{x}\ [\mathrm{mA}]} = \underbrace{\begin{bmatrix} 10 \\ 6 \end{bmatrix}}_{\mathbf{b}\ [\mathrm{V}]}$$

Expand the matrix product row by row:

$$\begin{bmatrix} 5I_1 + (-2)I_2 \\ (-2)I_1 + 7I_2 \end{bmatrix} = \begin{bmatrix} 10 \\ 6 \end{bmatrix}$$

The top row is Equation 1. The bottom row is Equation 2. If either row reads back differently from the KVL equation, fix the matrix before solving.

$$[\mathrm{k\Omega}][\mathrm{mA}] = [\mathrm{V}]$$

The unit choice makes the numerical answer come out in milliamperes.

6 Use the row rule to catch transcription errors

For each row, read the matrix back as an equation before solving:

$$[5,\ -2] \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = 10 \quad \Longrightarrow \quad 5I_1 - 2I_2 = 10$$

$$[-2,\ 7] \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = 6 \quad \Longrightarrow \quad -2I_1 + 7I_2 = 6$$

Stumbling block: changing the variable order in only one row creates a different problem. The solver may still return a clean-looking answer.

Solve two ways and verify

7 Numerical linear algebra

Use a solver for $A\mathbf{x} = \mathbf{b}$. Do not calculate A^{-1} by hand for this task.

```
import numpy as np

A = np.array([[5.0, -2.0],
              [-2.0, 7.0]])
b = np.array([10.0, 6.0])
x = np.linalg.solve(A, b)
residual = A @ x - b
```

8 Symbolic solution

Symbolic algebra keeps exact fractions instead of decimal approximations.

```
import sympy as sp

I1, I2 = sp.symbols("I1 I2")
equations = [sp.Eq(5*I1 - 2*I2, 10),
             sp.Eq(-2*I1 + 7*I2, 6)]
exact = sp.solve(equations, (I1, I2), dict=True)[0]
```

The exact solution is

$$I_1 = \frac{82}{31}\,\text{mA}, \qquad I_2 = \frac{50}{31}\,\text{mA}, \qquad I_3 = I_1 - I_2 = \frac{32}{31}\,\text{mA}.$$

9 Verify independently

$$\text{Equation 1:}\quad 5\left(\frac{82}{31}\right) - 2\left(\frac{50}{31}\right) = 10$$

$$\text{Equation 2:}\quad -2\left(\frac{82}{31}\right) + 7\left(\frac{50}{31}\right) = 6$$

$$\text{KCL:}\quad \frac{82}{31} - \frac{50}{31} = \frac{32}{31}$$

A near-zero residual proves that the calculated currents satisfy the equations entered into Python. It does not prove that the arrows, signs, units, or original circuit model were correct.

Part B: build the Question 5 matrix yourself

The goal: use the same method on Question 5 without being given its coefficients, completed matrix, or currents.

10 Define the currents and signs

In the notebook, answer before doing algebra:

1. What directions did you choose for I_1, I_2, and I_3?
2. Is your shared-current equation $I_3 = I_1 - I_2$ or $I_3 = I_2 - I_1$? Why?
3. Which source and resistor terms are rises? Which are drops?
4. Why does the shared-resistor sign differ between the two loop walks?

11 Show seven algebra lines for each loop

For **each** loop, show one move per line:

1. signed KVL using resistor-voltage symbols;
2. replace every resistor voltage with IR;
3. substitute the expression for I_3;
4. distribute through the bracket;
5. reorder into $[I_1, I_2]$;
6. combine like terms;
7. move the constant to the right side.

Your two boxed equations should have the structure

$$\boxed{a_{11}I_1 + a_{12}I_2 = b_1}, \qquad \boxed{a_{21}I_1 + a_{22}I_2 = b_2}.$$

12 Extract coefficients and prove the matrix matches

Question 5	coefficient of I_1	coefficient of I_2	right side
Equation 1	_____	_____	_____
Equation 2	_____	_____	_____

$$\underbrace{\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} I_1 \\ I_2 \end{bmatrix}}_{\mathbf{x}} = \underbrace{\begin{bmatrix} b_1 \\ b_2 \end{bmatrix}}_{\mathbf{b}}$$

Replace the six placeholders with your numbers. Read the top row back as Equation 1 and the bottom row as Equation 2. Only then use Jupyter to solve and compare with the released answer.

> The helper questions reveal the method, not the answer. Your algebra must generate the six numbers placed into A and $\mathbf{b}$.

Assignment and submission

Part A: show how the completed Question 4 circuit becomes two equations, one matrix, and two matching solver results.

Part B: use the helper questions to derive the Question 5 equations and matrix without copying the completed answer.

The essential skill: keep the shared-current definition and variable order consistent.

The challenge: show every signed term in both loop equations instead of copying the finished matrix.

The investigation: use Python help and the official NumPy or SymPy documentation when a function or object is unfamiliar.

The verification: use residuals, substitution, and $I_3 = I_1 - I_2$.

Required notebook evidence

1. Draw and label the directions of I_1, I_2, and I_3.
2. Show the three signed KVL terms for Loop 1 and derive Equation 1.
3. Show the three signed KVL terms for Loop 2 and derive Equation 2.
4. State the variable order and units.
5. Build A and $\mathbf{b}$, then read both matrix rows back as equations.
6. Solve with ‘numpy.linalg.solve‘ and report the residual vector.
7. Solve the same equations with ‘sympy.solve‘ and report exact fractions.
8. Calculate I_3 and compare all three currents with the released answer.
9. For Question 5, show both seven-line rearrangements, the coefficient table, the completed matrix, and both row read-backs.
10. Solve Question 5 only after the matrix is justified; then compare with the released answer.

Progressive hints

Hint 1 - concept: the shared resistor carries $I_1 - I_2$ because the mesh currents oppose each other there.
Hint 2 - documentation: search ‘numpy.linalg.solve A shape‘ or ‘SymPy solve simultaneous equations‘.
Hint 3 - technical: use rows $[5, -2]$ and $[-2, 7]$ only after you can read them back as the two standard-form equations.

Submit

Rename the notebook exactly:

`studentnumber-lastname-kirchhoff-jupyter.ipynb`

Example: `123456789-Smith-kirchhoff-jupyter.ipynb`. Keep the ‘.ipynb‘ ending. Restart the kernel, run all required cells, save, download, reopen, and submit the notebook through the assigned classroom system.

Later, not tomorrow: the intentional keyword error, array-shape error, unit bug, parameter sweep, capacitors, and RC response.

Help improve this lesson (optional)

Tell the teacher which step, sign, diagram, or code cell was unclear, what you still do not know how to do, and one change that would help the next student. A useful fix to the wording, schematic, example, or starter code may be added to the course materials and credited to you.